Partial Differential Equations, WIPDV-07 2012/13 semester II a
Examination, July 1, 2013

Name Student number

NOTE

- One hand-written A4 (double side) with notes is allowed.
- No printed A4 and/or A4 with notes copied from others is allowed.
- Only the use of standard numerical calculators is allowed.
- Write clearly all steps of the derivation and not only the final result.
- Total points=9 (one point will be added by default to the final grade). You should collect at least half of the total points (4.5 pts) to pass the exam.

END TIME: 17:00h

1. [pts 1] Find the domains where the following second order partial differential equations

$$u_{xx} + (1+y)^2\, u_{yy} = 0$$
$$y^2 u_{xx} - e^{2x}\, u_{yy} + u_x = 0$$

are elliptic, parabolic or hyperbolic, with x, y real.

2. [pts 3] Find the solution $u(x,t)$ of the inhomogeneous diffusion equation on the interval $(0,l)$

$$u_t - k u_{xx} = A\, \cos \omega t$$

$$u(x,0) = \phi(x) = \cancel{\cos \frac{\pi x}{l}} \quad \sin \frac{\pi x}{l}$$
$$u(0,t) = u(l,t) = \frac{A}{\omega} \sin \omega t \qquad t=0 \Rightarrow \text{they are all zero}$$

with the method of shifting the data. In particular, it is convenient to find the function $U(x,t)$ such that $v(x,t) = u(x,t) - U(x,t)$ satisfies the homogeneous diffusion equation $v_t - k v_{xx} = 0$.

Compute also explicitly the coefficients of the Fourier series for $v(x,t)$ and show that they are zero for $n > 0$ odd.

3. **[pts 3]** Find the harmonic function on the semi-infinite strip $\{0 \leq x \leq \pi,\ 0 \leq y < \infty\}$, with boundary conditions

$$\begin{aligned} &u(0,y) = u(\pi,y) = 0 \\ &u(x,0) = h(x) \qquad\qquad \lim_{y\to\infty} u(x,y) = 0 \end{aligned}$$

Solve it also for the specific case $h(x) = e^x$.

4. **[pts 2]** Show that the Dirichlet problem with the Poisson's equation

$$\begin{aligned} \Delta u &= f \qquad \text{in } D \\ u &= h \qquad \text{on bdy } D \end{aligned}$$

admits a unique solution, with the domain D a connected bounded open set, and u harmonic function in D and continuous on $\bar{D} = D \cup \text{bdy} D$.

Useful formulas:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$